# "Trashketball"

## Scatter Plots and Linear Inequalities

In this lesson, students toss trash-balls into the trashcan, recording their shooting percentage from various distances from the basket. With distances measured and percentages figured, students create a scatter plot and a line of best fit to create a linear model of the shooting skills of the class.

Students analyze various data points, first looking at one-variable inequalities and their graphs. Finally, students are led through a discussion to recognize the role that linear inequalities play. This is a great, simple and inexpensive lesson that addresses many significant Algebra 1 concepts including expressions, one- and two-variable inequalities and their graphs, data analysis, scatter plots, and linear equations.

### Lesson Snapshot

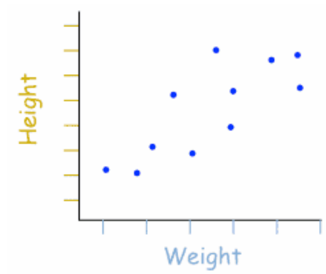
1. **Elicit**
   - **Formative Assessment Probe –** Students are presented a situation where they must interpret given data and predict outcomes.
2. **Engage**
   - **Shooting Percentage –** Students create a rule for determining the shooting percentage for a given sample.
   - **Trashketball –** Students gather data by creating a simple experiment where students take shots into a small trashcan from incremental distances, recording their shots made and missed.
3. **Explore**
   - **Creating a Mathematical Model –** Students begin to interpret their data in order to produce a mathematical model of the event using a line of best fit.
   - **Comparing Future Events to the Model with Inequalities –** Students are presented with a similar experiment, but with some type of interference or advantage. Students will select their modification and create appropriate inequalities for each situation.
4. **Explain**
   - **Inequalities vs. Linear Graphs –** Students are presented with an explanation of how linear graphs (or models) can provide a glimpse of exact data, but the linear inequality provides a larger possibility of solutions.
   - **Linear Inequalities and Their Solutions –** Students start seeing connections between the shaded area of the Linear Inequality Graphs and the solutions of Linear Inequalities.
5. **Elaborate and Evaluate**
   - **Back to Bowling –** In order to bring the conversation to a close, students now take a second look at the Formative Assessment Probe from the beginning of the lesson, piecing together much of what they've experienced during the lesson.
   - **Challenge Questions and Practice –** A set of challenge problems are posed to students that cause students to apply many facets of Algebra 1 while practicing their skills for solving and graphing inequalities.

## Materials

- 1 newspaper or used paper from class (enough for three or four trash balls)
- 1 standard commercial trashcan (http://bit.ly/ot8i4O)
- Tape measure if not on tiled floor and masking tape (or painter's tape)
- Rulers for each student
- Formative Assessment Probe (2 per student; beginning and ending of lesson)
- Trashketball Student Worksheet (1 per student; Exploration)

# Vocabulary Terms for the Teacher

- **Control (Variable) –** all other aspects of an experiment that must be held constant as to not interfere with the outcome of the experiment.
- **Correlation –** a relationship between two or more measures (or variables).
- **Dependent Variable –** the expected outcome of an experiment that is dependent upon the manipulated variable.
- **Hypothesis –** a statement that signifies the outcomes of an experiment based on various parameters.
- **Independent Variable –** an aspect of an experiment that is manipulated by the experimenter.
- **Line of Best Fit –** a type of line that approximates the correlation of data when represented on a scatter plot.
- **Linear Inequality –** a mathematical relationship involving a linear function such as $y = mx + b$. In linear inequalities, solutions may or may not lie on the line, where $y < mx + b$, $y > mx + b$, $y \le mx + b$, or $y \ge mx + b$.
- **Linear Model –** a mathematical function that algebraically represents a situation, in this case an equation of a line for the Line of Best Fit.
- **Negative Correlation –** a type of correlation in which the approximation of the correlation (Line of Best Fit) has a negative slope.
- **Positive Correlation –** a type of correlation in which the approximation of the correlation (Line of Best Fit) has a positive slope.
- **Relatively No Correlation –** a type of correlation in which their is no reasonable approximation for the data.
- **Scatter Plot –** A graph of plotted points that show the relationship between two sets of data. In this example, each dot represents one person's weight versus their height.




- **Shooting Percentage –** a measure of accuracy where

$$\frac{\text{shots made}}{\text{shots attempted}} \times 100 = \text{shooting percentage}$$

- **Slope –** the direction, or rate of change, of a line. $\frac{\text{rise}}{\text{run}} \_ \frac{\text{change in Y}}{\text{change in X}}$
- **Form –** a specific form of the equation of a line where $y = mx + b$.

  *m* represents the slope and *b* represents the y-intercept of a line.
- **Y-Intercept –** the point at which a line intersects the y-axis of a coordinate plane. The y-intercept does not exist for vertical lines.

# Student Objectives

### Students will…

- participate in an activity in order to gather and understand real-world data;
- develop and test a hypothesis;
- explore the relationship between independent and dependent variables;
- create data tables and use scatter plots to analyze correlations;
- organize data in order to develop a sense of domain and range;
- develop the equation of a line based on a line of best fit;
- create a deeper understanding of slope as it relates to the real-world;
- explore the accuracy of their linear model when variables of the experiment are modified;
- develop a conceptual understanding of linear inequalities;
- apply conceptual knowledge in practice problems.

# Pre-Requisite Knowledge for Students

### Students should be familiar with…

- slope as a rate of change;
- the equation for slope and how to solve the equation;
- solving equations for an unknown variable;
- creating a scatter plot;
- creating a line of best fit; and
- basic symbols associated with inequalities such as <, >, ≤, and ≥.

*These items will be utilized, but not covered in detail in this lesson.*

# Lesson Procedures [135 min. Total or 3 Class Periods]

## Elicit

### {Day 1}

### Formative Assessment Probe [5 min.]

*Students are presented a situation where they must interpret given data and predict outcomes.*

Begin the lesson with the **Formative Assessment Probe** by clarifying to all students that the worksheet will not be used for credit, but is important to the learning process.

- Be sure to work with students to help them clearly explain their reasoning for marking or not marking each letter.
- Have students use the back of the sheet if necessary.

❖ If you have low-performing writers, have them speak their explanation and help them to write down what they are saying.

**Gather all probes** and save them so that you can:

(1) read through student comments to better understand what misconceptions they may have and
(2) have them to compare to when you give them the probe again at the end of the lesson.

**A**, **C**, and **E** should be marked. See the Answer Key for details regarding each statement.

# Engage

## Shooting Percentage: [5 min.]

*Students create a rule for determining the shooting percentage for a given sample.*

**Create a common understanding of shooting percentage through a short warm up.**

- Ask students to come up with a simple definition of shooting percentage.
- Have students build a classroom definition on the board.
- Once students agree on the definition, have students consider a mathematical definition that would be useful in determining the shooting percentage of any situation.
- *Teacher assistance will vary according to the experience of the students.*
    - ***Example***: "Shooting percentage can be found by creating a ratio of the number of '*shots made*' to the number of '*shots attempted*.' Multiply this ratio by 100 to create a percent."
    - This can be written algebraically where shots attempted is ***a*** and shots made is ***m***.

      $$\frac{m}{a} \times 100 = \text{Shooting Percentage}$$

- With the definition agreed upon, **present students with 2 to 5 problems to give them practice** solving for the shooting percentage. A few examples and solutions are provided below:
  1. 5 attempts, 3 shots made
     a. $\frac{3}{5} \times 100 = .60 \times 100 = 60\%$
  2. 9 attempts, 8 shots made
     a. $\frac{8}{9} \times 100 = .88 \times 100 = 88\%$
  3. 4 attempts, 0 shots made (*Discuss that 0 out of any number of attempts is 0%.*)
     a. $\frac{0}{4} \times 100 = 0 \times 100 = 0\%$
  4. 7 attempts, 7 shots made (*Discuss that perfect shooting percentage is 100%.)*
     a. $\frac{7}{7} \times 100 = 1.00 \times 100 = 100\%$
  5. 12 attempts, 13 shots made (*Discuss how this situation is impossible since you cannot make more shots than you took.)*
     a. $\frac{13}{12} \times 100 = 1.083 \times 100 = 108.3\%$

## Trashketball [15 min.]

*Students gather data by creating a simple experiment where students take shots into a small trash can from incremental distances, recording their shots made and missed.*

With the understanding of shooting percentage developed, lead the class to **create a hypothesis** that describes the relationship, if any, between the distance from the basket and the shooting percentage.

- *This is included on the Trashketball Handout listed in the Materials section.*

Introduce the concept of Trashketball to the class. Use old newspaper or scrap paper to create a few nearly identical wads of trash. Take a few shots to demonstrate the appropriate shooting technique and retrieving process.

- Ask students to consider if and how moving away from the basket will affect the shooting percentage.

Students should determine whether there **IS** or **IS NOT** an effect.

- If they decide that there **IS** an effect, they should determine if the shooting percentage will **INCREASE** or **DECREASE**.

Have students create a hypothesis, for Question 1 of the Trashketball Handout, based on their belief.

- Sample hypotheses are provided here*:*
  - **IS NOT**: As the distance from the basket increases, there will be no change in the shooting percentage.
  - **IS – INCREASE**: As the distance from the basket increases, the shooting percentage will increase.
  - **IS – DECREASE**: As the distance from the basket increases, the shooting percentage will decrease.
- Allow students a few moments to discuss the **Independent** and **Dependent variables**.
  - Emphasize considering which variable *depends* on the other variable.
  - Have students record their responses on Questions 2 and 3 of the Trashketball Handout.
- Have the class brainstorm the **Control variables** of the experiment.
  - Have students record their responses on Question 4 of the Trashketball Handout.

- *More information about Independent, Dependent, and Control variables can be found in the "Vocabulary Terms for the Teacher" section on Page 2. Also, possible answers for each of the variables can be found on the Trashketball Handout Answer Key.*

Before beginning to collect data using the Trashketball Handout, create groups of 4 or 5.
Have each group assigned to two distances on the table. Allow each member of the group to shoot about 3 times so that there are between 12 and 15 attempts at each distance.

- *It may be a good idea to let the group that shoots at 0 ft. also shoot at 16 ft., or the largest distance you choose to include in your data table.*

Have the entire class record the data on Question 5 of their own Trashketball Handout.
Once all data is collected, have students work out the Shooting Percentages for the entire table. Space is provided on the Trashketball Handout, but students should be encouraged to utilize their calculators to complete the work quickly as it is not meant to be the primary focus of the lesson.

- *If students are still struggling with the application of the Shooting Percentage algorithm, please be sure to make note of this and consider implementing remediation strategies for that student.*

Have students read their Shooting Percentages aloud so that other members of the class can self-check.
Go over inconsistencies at the front of the class.

# Explore

## Creating a Mathematical Model [20 min.]

*Students begin to interpret their data in order to produce a mathematical model of the event using a line of best fit.*

Utilizing the Trashketball Handout as a guide, students now take their data and translate it into a scatterplot on Question 6.

- ❖ *Although correlation is not a major part of this lesson or Algebra 1 standards, it plays an important piece in connecting slope to real data. Introducing positive and negative correlation here provides another set of tools for students to consider the relationship of data when looking at a scatter plot. Results of this instruction should be seen on the Formative Assessment Probe, statement D.*

Present to students the four definitions for *correlation*, *positive correlation*, *negative correlation*, and *relatively no correlation*, found in the "Vocabulary Terms for the Teacher" section on Page 2. Have students record the definitions on Questions 7-10 of the Trashketball Handout.

- Ask students to answer Questions 11 and 12 silently.
- Engage all students in a class discussion regarding how they were able to define their scatter plot as correlated positively, negatively, or not at all.
- Bring the class to the consensus that the data, although it likely maintains a high Shooting Percentage for the first few data points, still has a Negative Correlation.
- Students should also consider how the Negative Correlation lends itself best to the **IS – DECREASE** hypothesis structure seen on Page 5.

With the Negative Correlation in mind, have students create a Line of Best Fit on Question 6 by drawing a line through at least two data points so that there are equal numbers of data points above and below the line.

- Have students utilize the Trashketball Handout to quickly create a Linear Model for the Line of Best Fit. The Handout guides students, step-by-step through the process on Questions 14-17.
- If your students do not need the step-by-step guide, or you would like to challenge them, you might have them put away their Handout until they have completed the Linear Model.

- ❖ *Remember that the Answer Key for the Trashketball Handout is provided, but the data will be unique to each class.*

## {Day 2}

## Comparing Future Events to the Model with Inequalities [15 min.]

*Students are presented with a similar experiment, but with some type of interference or advantage. Have students select their modification and create appropriate inequalities for each situation.*

Have students in each group choose a method by which they will adjust only one Control.

- Some examples of these modifications might be blindfolds, a fan blowing across the top of the trashcan, a larger/smaller trashcan, a larger/smaller trash ball, and there are many more.
- List the selected Manipulated Variable on Question 18.
- Considering that students have created a model for the unencumbered shooting experience, students now will argue whether the manipulated experience will cause the Shooting

Percentages to be less than (<), less than or equal to (≤), greater than (>), or greater than or equal (≥)to what the model would suggest.

- In other words, if the original Linear Model from Question 17 is y = -5x + 100 and the group selected *blindfolding* as a way to interfere with the Control variable of "Clear vision of the trashcan," the group might agree that their Shooting Percentage will definitely be lower than before. They would create the Linear Inequality y < -5x + 100.
- Have students record their Linear Inequality on Question 20.

- Have students test their new Linear Inequality by gathering new data using their Manipulated Variables.
- Students should record their data on the table on Question 21.

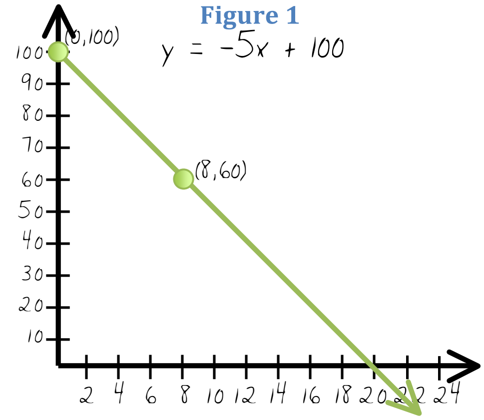
# Explain

## Inequalities vs. Linear Graphs [15 min.]

*Students are presented with an explanation of how linear graphs (or models) can provide a glimpse of exact data, but the linear inequality provides a larger possibility of solutions.*

Before students complete Question 22, first explain that they are going to look at an example to help them make sense of what they've just done.

**Figure 1**




- Consider the graph of the equation y = -5x + 100 shown in **Figure 1**.
- The equal sign tells us that there is some type of exact relationship between the *x* and the *y*. In this case, you can choose any *x*-value, multiply it by -5 and add 100 to find the *y*-value.
- This can be seen with the data point (8,60)
- -5(8) + 100 = -40 + 100 = 60
- This relationship is strict and can be called "one-to-one" since each *x*-value has only one *y*-value. This Is also known as a **function**.

Now, consider the situation where a person is blindfolded before shooting. It is natural to expect the resulting Shooting Percentage to be less than the current model of the unobstructed shot. We can use our Linear Model and change the "=" to "<" (less than) since we believe that the results will be lower. If we thought the Shooting Percentage would improve, we would change the "=" to ">" (greater than).

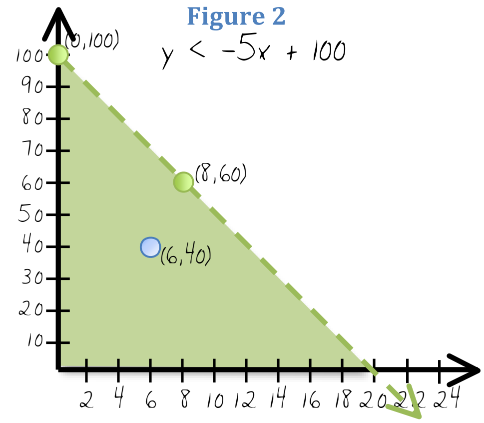
What does the Linear Inequality y < -5x + 100 look like? Only two things have to be considered when converting a Linear Equation to a Linear Inequality.

(1) The line is dotted if it is < or >.
(2) The graph is shaded above (> or ≥) or below (< or ≤) the line.

Since we have y < -5x + 100, we can recognize that:

(1) The line will be dotted.
(2) The graph will be shaded below the line.

Figure 2




The new Linear Inequality no longer represents a strict relationship between *x* and *y*. Before discussing the significance of the dotted or solid line and the shading, have students complete their own Linear Inequalities on Question 22.

With their data points plotted, open up discussion on the significance of the data falling in the shaded area of their graph.

## Linear Inequalities and Their Solutions [15 min.]

*Students start seeing connections between the shaded area of the Linear Inequality Graphs and the solutions of Linear Inequalities.*

Explain to students that Linear Inequalities are much different from Linear Equations in that they represent an entire half plane (excluding the line in this case). There are a multitude of (x,y) pairs that can be found to create a true statement.

- Let's look at the example of (6,40)

  40 < -5(6) + 100 Substitution Property of Inequality.
  40 < -30 + 100 Simplification.
  40 < 70 Simplification.

- We would expect this to be a true statement since the coordinate point (6,40) falls in the shaded area as seen in **Figure 2**.

If we want to find a range of *y*-values that satisfy the Linear Inequality for a certain *x*-value, we can not only use the Algebraic Properties to solve the inequality, but the single-variable inequality actually represents a line graph.

- Let's use the same Linear Inequality and assume that x = 14.

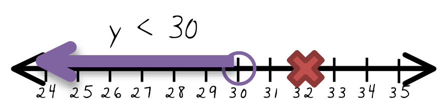
  y < -5x + 100 The original inequality.
  y < -5(14) + 100 Substitution Property of Inequality.
  y < -70 + 100 Simplification.
  y < 30 Simplification.

- This result, y < 30, tells us that any coordinate pair where x = 14 and the y < 30 will fall in the shaded area of the graph seen in **Figure 2** and will be a solution to the inequality y < -5x + 100.

To graph y < 30,

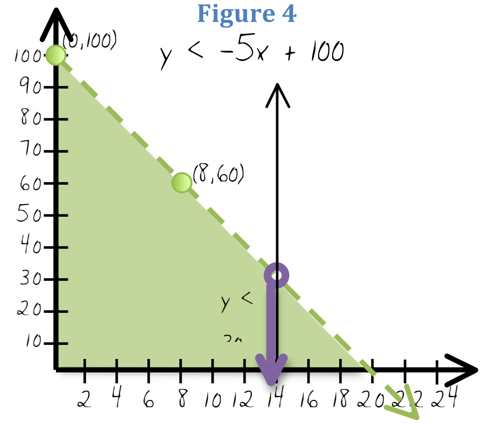
(1) Draw a number line that includes 30.
(2) Place an open circle (○) on the 30. If the inequality were ≤ or ≥, it would be a closed circle (●).
(3) To determine which direction to shade, consider one of the numbers on the number line and determine whether it satisfies the inequality. For example, if you choose 32, 32 < 30 is not true, so you would shade away from 32.
(4) See **Figure 3**.

Figure 3



An interesting way to think about this single-variable inequality is in the context of the original Linear Inequality. In **Figure 4**, the single-variable inequality and the Linear Inequality have been merged to show that when x = 14, the values for y must be less than 30.

**Figure 4**




To be sure that the point is clear you might ask students to solve the inequality y < -5x + 100 when y = 60.

- ❖ *This problem only requires that the students look at the graph. The solution is obvious since we know that when y = 60, x = 8. We also know that the x-values are shaded to the left. Therefore, the solution is x < 8.*

# Elaborate and Evaluate

## {Day 3}

## Back to Bowling [5 min.]

*In order to bring the conversation to a close, students now take a second look at the Bowling Probe from the beginning of the lesson, piecing together much of what they've experienced during the lesson.*

To begin wrapping up the lesson, the **Formative Assessment Probe** will help clarify what the students have learned. This worksheet should be used as a small grade, perhaps assigning values from 0-4, where the following is true:

0 – Incomplete
1 – The student has no correct marks and/or no explanation.
2 – The student has invalid explanations.
3 – The student has not selected each of the three correct statements and/or has incomplete reasoning. The student may have also select statements that were untrue.
4 – The student has correctly selected each of the 3 statements and has appropriately clarified their reasoning.

- Have students use the back of the handout if necessary.

❖ If you have low-performing writers, have them speak their explanation and help them to write down what they are saying.

**Gather all probes**. **A**, **C**, and **E** should be marked. See the Answer Key for details regarding each statement.

Discuss the correct statements and explanations through a class wide discussion.

## Challenge Questions and Practice [40 min.]

*A set of challenge problems are posed to students that cause students to apply many facets of Algebra 1 while practicing their skills for solving and graphing inequalities.*

**Challenge 1:** Describe the differences in the graphs of x ≤ -2 and x > -2.

**Challenge 2:** During normal years, the depth that a water well is drilled (x = feet) is related to the water (y = 1 gallon/hour) produced. The equation can be described by the Linear Equation $y = 3x – 75$. Create a Linear Inequality that describes the water production during a particularly rainy year.

**Challenge 3:** Based on the created Linear Inequality from #2, is it appropriate to expect to produce 100 gallons/hour at the depth of 55 feet? Defend your answer with appropriate graphs and inequalities.

**Challenge 4:** Based on the created Linear Inequality from #2, is it appropriate to expect to produce 200 gallons/hour at the depth of 100 feet? Defend your answer with appropriate graphs and inequalities.

**Challenge 5:** Create your own Linear Inequality. Sketch a graph of it and solve it for a value of x and a value of y that you choose. Graph each solution on its own number line.

- ❖ *Although it is not included here, it is important to provide students more practice and ample time to have questions answered and misunderstanding clarified. This could likely take 1.5 class periods leaving the remaining portion of the week for a Quiz that connects students back to high-stakes exam-style questions. If you create a quiz that you would like to share, please visit [http://k20alt.ou.edu/groups/trashketball](http://k20alt.ou.edu/groups/trashketball).*

# PASS

## Content Standards

- **Standard 2.1a –** Distinguish between linear and nonlinear data.
- **Standard 2.1c –** Identify dependent and independent variables, domain and range.
- **Standard 2.2d –** Develop the equation of a line, and graph linear relationships given two points on the line.
- **Standard 2.3a –** Solve linear inequalities by graphing or using properties of inequalities.
- **Standard 2.3b –** Match inequalities (with 1 or 2 variables) to a graph, table, or situation and vice versa.
- **Standard 3.2 –** Collect data involving two variables and display on a scatter plot; interpret results using a linear model/equation and identify whether the model/equation is a line of best fit for the data.

## Process Standards

- **Standard 4.1 – Connections:** Link mathematical ideas to the real world.
- **Standard 5.2 – Representation:** Use a variety of mathematical representations as tools for organizing, recording, and communicating mathematical ideas.

# Common Core State Standard

*For more information go to http://corestandards.org/*

## Algebra – Creating Equations [A-CED]

### Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

## Algebra – Reasoning with Equations and Inequalities [A-REI]

### Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

### Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## Functions – Interpreting Functions [F-IF]

### Interpret functions that arise in applications in terms of the context

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

## Functions – Linear and Exponential Models [F-LE]

### Construct and compare linear and exponential models and solve problems

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

### Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.